$c_{0j}+u_j^2 \geq 0$ for all objects j.

$c_{ij}+u_i^{\prime}+u_j^2 = 0$ for all $(i,j) \in S$.

$c_{ij}+u_j^2 \leq _k^{min}\{c_{ik}+u_k^2\}$ for all $(i,j) \in S$.

Thus we can prove the following proposition.

Proposition: If we assume that $c_{0j}+u_j^2 \geq 0$ at the start of the Forward Auction Algorithm and all of the persons are assigned via a forward step, then we have:

$c_{ij}+u_i^{\prime}+u_j^2 \geq -\epsilon$ for all $(i,j) \in A$.

$c_{ij}+u_i^{\prime}+u_j^2 = 0$ for all $(i,j) \in S$.

$c_{ij}+u_j^2 \leq _k^{min}\{c_{ik}+u_k^2\}+\epsilon$ for all $(i,j) \in S$.

Optimality of the Algorithm

Theorem: ϵ-CS preserved during every forward and reverse iteration.

Theorem: If a feasible solution exists, then the resulting solution is with $m\epsilon$ of being optimal for the Combined Forward Reverse Algorithm.

Implementation Specifics

Parallelization

Here are but a few comments.

Although the algorithm appears to be serial in nature, its primary computational requirements are almost entirely parallelizable. Thus parallelization is planned.

Step 2 is the computational part of the algorithm. Evaluating $\Phi_{N-k+1}\chi\chi\chi$ procedure requires 99% of the computing time in the algorithm. This part uses two dimensional assignment algorithms, a search over a large number of indices, and a nonsmooth optimization algorithm. It is the second part (the search) that consumes 99% of the computational time and this is almost entirely parallelizable. Indeed, there are two dimensional assignment solvers that are highly parallelizable. Thus, we need but parallelize the nonsmooth optimization solver to have a reasonably complete parallelization.

If a sensitivity analysis is desired or if one is interested in computing several near-optimal solutions, a parallel processor with a few powerful processors and good communication such as on the Intel Paragon would be most beneficial.

The foregoing discussion of the invention has been presented for purposes of illustration and description. Further, the description is not intended to limit the invention to the form disclosed herein. Consequently, variation and modification commensurate with the above teachings, within the skill and knowledge of the relevant art, are within the scope of the present invention. The embodiment described hereinabove is further intended to explain the best mode presently known of practicing the invention and to enable others skilled in the art to utilize the invention as such, or in other embodiments, and with the various modifications required by their particular application or use of the invention. It is intended that the appended claims be construed to include alternative embodiments of the invention to the extent permitted by the prior art.

What is claimed is:

1. A method for tracking a plurality of objects, comprising:

repeatedly scanning a region containing a set consisting of one or more moving objects and generating N sequential images or data sets of said region, a plurality of observations in said images or data sets providing positional information for objects in said set;

determining a plurality of tracks, at least one track for each object in said set;

determining a plurality of costs, wherein each cost is for assigning one of said observations to one of said tracks;

defining a linear programming problem:

$$\text{Minimize} \quad \sum_{i_1 \cdots i_N} c_{i_1 \cdots i_N} z_{i_1 \cdots i_N}$$

$$\text{Subject To} \quad \sum_{i_2 i_3 \cdots i_N} z_{i_1 \cdots i_N} = 1 \quad (i_1 = 1, \ldots, M_1)$$

$$\sum_{i_1 i_3 \cdots i_N} z_{i_1 \cdots i_N} = 1 \quad (i_2 = 1, \ldots, M_2)$$

$$\sum_{i_1 \cdots i_{p-1} i_{p+1} \cdots i_N} z_{i_1 \cdots i_N} = 1$$

$$(i_p = 1, \ldots, M_p \text{ and } p = 2, \ldots N-1)$$

$$\sum_{i_1 i_2 \cdots i_{N-1}} z_{i_1 \cdots i_N} = 1 \quad (i_N = 1, \ldots, M_N)$$

$$0 \leq z_{i_1 \cdots i_N} \leq 1 \text{ for all } i_1, \ldots, i_N,$$

wherein each $c_{i_1 \cdots i_N}$ is included in said plurality of costs, each M_i, i=1, ...N, being one of: (a) a number of observations in an ith image or data set of said N sequential images or data sets; (b) a sum of a number of tracks in said plurality of tracks, and a number of said observations in the ith image or data set not assigned to one of said tracks; and (c) a number of tracks in said plurality of tracks;

solving said linear programming problem for values of $z_{i_1 \cdots i_N}$ for each i1 . . . iN;

determining a value $z_{i_1 \cdots i_N}$ in $\{0,1\}$ for each i1 . . . iN corresponding to each $z_{i_1 \cdots i_N}$, wherein said values zi1 . . . iN provide an optimal or near optimal solution to said linear programming problem;

taking one or more of the following actions based on said optimal or near-optimal assignment of said plurality of points to said plurality of tracks:

sending a warning to aircraft or a ground or sea facility,

controlling air traffic,

controlling anti-aircraft or anti-missile equipment,

taking evasive action,

working on one of said one or more objects, surveilling one of said one or more objects.

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